

2008 High School Math Contest
Draft #3

Elon University

April 12, 2008

Note : In general, figures are drawn *not to scale!* All decimal answers should be rounded to two decimal places.

1. On average, how often does the minute hand pass the hour hand on an ordinary clock.
 - (a) 60.2 minutes.
 - (b) 60.73 minutes.
 - (c) 61.3 minutes.
 - (d) 64 minutes.
 - (e) 65.45 minutes.
2. Find the product of the solutions for $x^4 - 18x^3 - 25x^2 + 114x = 0$.
 - (a) -114 .
 - (b) -6 .
 - (c) 6 .
 - (d) 114 .
 - (e) None of these.
3. Determine the number of positive divisors of 2008.
 - (a) 6.
 - (b) 8.
 - (c) 9.
 - (d) 12.
 - (e) 16.
4. Point P is 9 units from the center of a circle of radius 15. How many different chords of the circle contain P and have integer lengths?
 - (a) 11.
 - (b) 12.
 - (c) 13.
 - (d) 14.
 - (e) 15.

5. A positive integer N is a *palindrome* if the integer obtained by reversing the sequence of digits of N is equal to N . The year 1991 is the only year in the past century with the following two properties:
- It is a palindrome.
 - It factors as a product of a 2-digit prime palindrome and a 3-digit prime palindrome.

How many years in the millenium between 1000 and 2000 (including 1991) have these two properties?

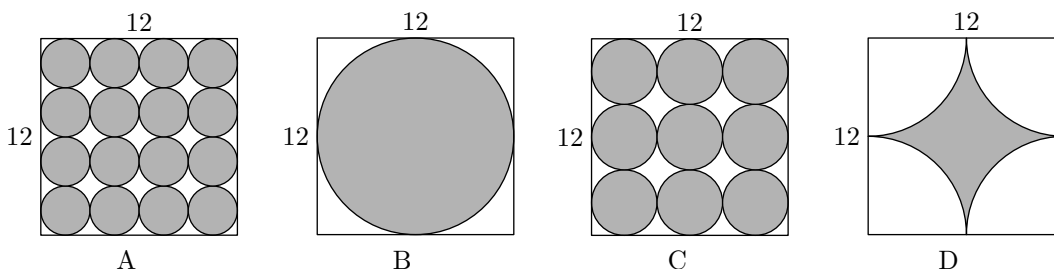
- (a) 1.
 (b) 2.
 (c) 3.
 (d) 4.
 (e) 5.
6. Find x .

$$x = 3 + \frac{1}{6 + \frac{1}{6 + \frac{1}{6 + \frac{1}{\dots}}}}$$

- (a) $\frac{79}{25}$.
 (b) $\sqrt{10}$.
 (c) $\frac{253}{80}$.
 (d) $\frac{3}{2}\sqrt{5}$.
 (e) $2\sqrt{3}$.
7. The front wheels of a wagon measure 3.5 feet in diameter. The rear wheels measure 4.25 feet in diameter. While the wagon is stopped, somebody makes a chalk mark on both a front and a rear wheel. How far must the wagon travel before both chalk marks return to their initial positions at the same time?
- (a) 11.78 feet.
 (b) 18.64 feet.
 (c) 186.92 feet.
 (d) 194.71 feet.
 (e) 273.48 feet.
8. Of all the combinations of positive integers that sum to 10, what is their biggest possible product?
- (a) 20.
 (b) 24.
 (c) 28.
 (d) 30.
 (e) 36.

9. Let $f(x) = x^2 + x$, $f_1(x) = f(x)$, and $f_{n+1}(x) = f(f_n(x))$ for $n \geq 1$. How many real roots does $f_{10}(x)$ have?
- (a) 0.
 (b) 1.
 (c) 2.
 (d) 20.
 (e) 1024.

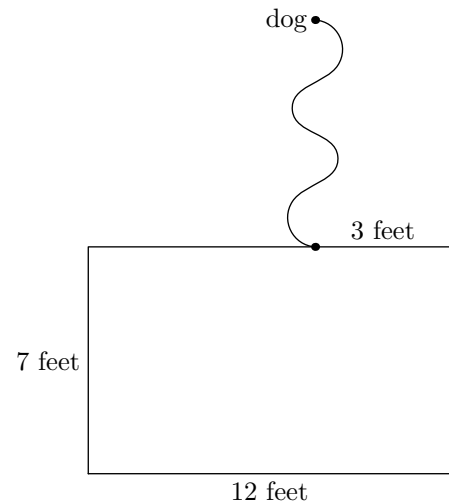
10. Which of the following shaded regions has the greatest area?



- (a) A.
 (b) B.
 (c) C.
 (d) D.
 (e) None of these.
11. To satisfy the equation $\frac{a+b}{a} = \frac{b}{a+b}$, a and b must be:
- (a) both rational.
 (b) both real, but not rational.
 (c) both not real.
 (d) one real, one not real.
 (e) one real, one not real OR both not real.
12. 5 cows can eat 2 acres of grass in 10 days. 7 cows can eat 3 acres of grass in 30 days. The grass grows at a constant rate and cows eat the grass at a constant rate. The length of the grass before the cows begin grazing is constant. How many days will it take 16 cows to eat 7 acres of grass?
- (a) 45 days.
 (b) 49 days.
 (c) 54 days.
 (d) 70 days.
 (e) 87 days.
13. A really old record turns at 78 rpm (revolutions per minute) and is 12 inches in diameter. How fast does a point on the outside edge of the record travel?
- (a) 1 foot per minute.
 (b) 118.4352 feet per minute.
 (c) 236.8704 feet per minute.
 (d) 245.0442 feet per minute.
 (e) none of the above.

14. My dog is staked to a point 3 feet from the corner of my 12×7 shed as shown to the right. His chain is 13 feet long. Supposing he pulls tightly on the chain, through what area can his leash sweep?

- (a) $\frac{169}{2}\pi$ square feet.
 (b) $\frac{463}{4}\pi$ square feet.
 (c) $\frac{269}{2}\pi$ square feet.
 (d) 169π square feet.
 (e) 227π square feet.



15. Two poker chips are identical, with each side colored either red or black. How many ways are there to color them if we consider two colorings to be the same if they involve switching the chips and/or flipping one or both chips?

- (a) 6.
 (b) 8.
 (c) 12.
 (d) 16.
 (e) 32.

16. If $\log_{2x} 216 = x$, where x is real, then x is:

- (a) a non-square, non-cube integer.
 (b) a non-square, non-cube, non-integral rational number.
 (c) an irrational number.
 (d) a perfect square.
 (e) a perfect cube.

17. A *great circle* on a sphere is one with the same center as the center of the sphere. The shortest distance between two points on a sphere is along an arc that is part of a great circle.

Assume that the surface of the Earth is a sphere. A triangle is drawn on the surface with one vertex at the North Pole, and the other two vertices on the Equator. The sides are arcs of great circles.

If the area of the triangle is $(1/48)$ th of that of the Earth's surface, then what is the sum of the angles in the triangle?

- (a) 180° .
 (b) 185° .
 (c) 190° .
 (d) 195° .
 (e) 200° .

18. Let $(1, 2)$, $(2, 3)$, $(3, 6)$, and $(4, 11)$ be points in the xy -plane. Let $p(x)$ be the polynomial of smallest degree whose graph goes through all four points. Find the sum of the leading coefficient of $p(x)$ with its degree.
- (a) 3.
 - (b) 4.
 - (c) 5.
 - (d) 6.
 - (e) 7.
19. Triangle T has vertices $(0, 38)$, $(2, 0)$, and $(38, 0)$. Circle C with radius r circumscribes T . Which of the following is closest to r ?
- (a) 24.
 - (b) 25.
 - (c) 26.
 - (d) 27.
 - (e) 28.
20. What is the most pieces you can form with n straight cuts across the face of a pizza?
- (a) $n(n - 1)$.
 - (b) n^2 .
 - (c) $n \left(\frac{n + 1}{2} \right) + 1$.
 - (d) $n^2 - 4$.
 - (e) $n \left(\frac{n + 1}{2} \right)$.
21. An acute isosceles triangle, $\triangle ABC$, is inscribed in a circle. Through B and C , tangents to the circle are drawn, meeting at point D . If $\angle ABC = \angle ACB = 2\angle D$ and X is the radian measure of $\angle CAB$, the $X =$
- (a) $\frac{3}{7}\pi$.
 - (b) $\frac{4}{9}\pi$.
 - (c) $\frac{5}{11}\pi$.
 - (d) $\frac{6}{13}\pi$.
 - (e) $\frac{7}{15}\pi$.
22. A student on vacation for d days observed that (1) it rained 7 times, morning or afternoon (2) when it rained in the afternoon, it was clear in the morning (3) there were five clear afternoons (4) there were six clear mornings. Then d equals:
- (a) 7.
 - (b) 9.
 - (c) 10.
 - (d) 11.
 - (e) 12.

23. Find the area of the region enclosed by the graph of $|x| + |y| = 6$.
- (a) 18.
 - (b) 36.
 - (c) 72.
 - (d) 144.
 - (e) None of these.
24. Each day a man meets his wife at the train station after work and then she drives him home. She always arrives exactly on time to pick him up. One day he catches an earlier train and arrives at the station one hour earlier. He immediately begins walking home along the same route his wife drives. Eventually, his wife sees him on her way to the station and drives him the rest of the way home. When they arrive home, the man notices that they arrived 20 minutes earlier than usual. How much time did the man spend walking?
- (a) 37 minutes.
 - (b) 40 minutes.
 - (c) 41 minutes.
 - (d) 42 minutes.
 - (e) 50 minutes.
25. If $(a + \frac{1}{a})^2 = 3$, then $a^3 + \frac{1}{a^3}$ equals:
- (a) 0.
 - (b) $3\sqrt{3}$.
 - (c) $\frac{10\sqrt{3}}{3}$.
 - (d) $6\sqrt{3}$.
 - (e) $7\sqrt{3}$.
26. A subset of the integers $1, 2, 3, \dots, 99, 100$ has the property that none of its members is 3 times another. What is the maximum number of members such a subset can have?
- (a) 50.
 - (b) 66.
 - (c) 67.
 - (d) 76.
 - (e) 78.
27. A boy flying a kite has played out all 100 feet of line. He notices that the line (which is perfectly straight) makes an angle of $\frac{\pi}{3}$ radians with the ground. How high is the kite?
- (a) 50 feet.
 - (b) 86.6 feet.
 - (c) 100 feet.
 - (d) 173.21 feet.
 - (e) none of the above.

28. If S is the set of points z in the complex plane such that $(3 + 4i)z$ is a real number, then S is a
- (a) right triangle.
 - (b) circle.
 - (c) hyperbola.
 - (d) line.
 - (e) parabola.
29. Person A can run around a track in 40 seconds. Person B , running in the opposite direction, meets Person A every 15 seconds. What is B 's time to run around the track, expressed in seconds?
- (a) 12.5.
 - (b) 24.
 - (c) 25.
 - (d) 27.5.
 - (e) 55.
30. Two six-sided dice are constructed to not be fair. Each is labeled with 1, 2, 3, 4, 5, and 6. The chance of rolling a given number on a single die is proportional to that number. Find the probability of rolling both dice (independent of each other) and getting a sum of 8.
- (a) 0.16.
 - (b) 0.17.
 - (c) 0.18.
 - (d) 0.19.
 - (e) 0.20.
31. The Puzzleville dating service has m men and w women, with $m > w$. Introductions are made by randomly drawing any two names (without replacement) out of the total membership. If one man and one woman are drawn, then a blind date is arranged. The probability that a draw will result in one of each gender is exactly 50%. After an advertising campaign to attract women to the club, 100 more members join the service. After the increase in membership, both properties still hold true: $m > w$ and a 50% probability of a mixed gender couple drawn. After the increase, how many men and women are in the service?
- (a) 315 men, 276 women.
 - (b) 300 men, 276 women.
 - (c) 179 men, 108 women.
 - (d) 275 men, 195 women.
 - (e) 350 men, 271 women.

32. An n -digit positive integer is *cute* if its n digits are an arrangement of the set $\{1, 2, 3, \dots, n\}$ and its first k digits form an integer that is divisible by k for $k = 1, 2, \dots, n$. For example, 321 is a cute 3-digit number integer because 1 divides 3, 2 divides 32 and 3 divides 321. How many cute 6-digit integers are there?
- (a) 0.
 - (b) 1.
 - (c) 2.
 - (d) 3.
 - (e) 4.
33. A rubber ball is dropped from a height of one meter, and bounces back up and continues to bounce up and down. Each time it bounces it returns to $\frac{2}{3}$ the height from whence it fell. What is the total distance traveled by the ball in meters?
- (a) 3.
 - (b) 4.
 - (c) 5.
 - (d) 6.
 - (e) 10.
34. In a group of students, 60% are wearing red shoes and 20% are wearing both red shoes and a green hat. If 10% are wearing neither red shoes nor a green hat and 12 students are wearing a green hat and not wearing red shoes, how many students are there?
- (a) 12.
 - (b) 30.
 - (c) 40.
 - (d) 100.
 - (e) none of the above.
35. The sides of a triangle are 25, 39, and 40. The diameter of the circumscribing circle is:
- (a) $\frac{125}{3}$.
 - (b) $\frac{133}{3}$.
 - (c) 40.
 - (d) 41.
 - (e) 42.