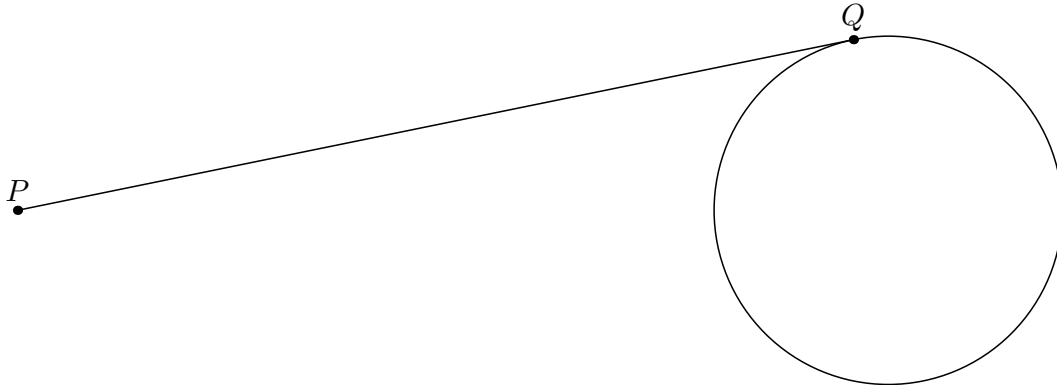


High School Mathematics Contest

Elon University Mathematics Department

Saturday, April 12, 2014

1. The point P is 10 cm away from the center of the circle which has radius 2 cm. The line \overleftrightarrow{PQ} is tangent to the circle.



What is the length of the line segment \overline{PQ} , rounded to the nearest tenth of a centimeter?

- (a) 9.4 cm.
 (b) 9.5 cm.
 (c) 9.6 cm.
 (d) 9.7 cm.
 (e) 9.8 cm.
2. $\frac{1}{1 - \sqrt[3]{2} + \sqrt[3]{4}}$ is equivalent to
- (a) $1 + \sqrt[3]{2}$.
 (b) $\frac{1}{3} - \frac{1}{3}\sqrt[3]{2}$.
 (c) $-\frac{1}{3} + \frac{1}{3}\sqrt[3]{2}$.
 (d) $-1 + \sqrt[3]{2}$.
 (e) $\frac{1}{3} + \frac{1}{3}\sqrt[3]{2}$.
3. Painters A and B can paint a wall in 10 hours when working at the same time. Painter B works twice as fast as A. How long would it take painter B to paint it if he worked alone?
- (a) 18 hours.
 (b) 17.2 hours.
 (c) 20 hours.
 (d) 15 hours.
 (e) 30 hours.
4. Let c be a positive integer and suppose $x_1 = 3$ and $x_2 = 7$ with $x_{n+1} = cx_n + (c+1)x_{n-1}$ for $n \geq 2$. It can be shown that $\lim_{n \rightarrow \infty} \frac{x_{n+1}}{x_n}$ exists for all values of $c > 0$. How many values of c are there so that $\lim_{n \rightarrow \infty} \frac{x_{n+1}}{x_n}$ is an integer?
- (a) 0.
 (b) 3.
 (c) 7.
 (d) 15.
 (e) More than 15.

5. For how many values of n (where n is an integer greater than one) is subtraction, defined on the integers modulo n , associative?
- (a) None.
 - (b) One.
 - (c) Two.
 - (d) Three.
 - (e) Infinitely many.
6. What is the radius of a sphere circumscribed about a cube, with each edge measuring 3cm ?
- (a) $\frac{3\sqrt{3}}{2}$.
 - (b) $3\sqrt{3}$.
 - (c) $3\sqrt{2}$.
 - (d) $\frac{3\sqrt{2}}{2}$.
 - (e) 5.
7. Let $ABCD$ be a trapezoid with the measure of AB twice that of base DC , and let E be the point of intersection of the diagonals. If the measure of diagonal AC is 11, find the measure of segment EC .
- (a) $11/2$.
 - (b) $\sqrt{125}$.
 - (c) $11/3$.
 - (d) $\sqrt{130}$.
 - (e) $\sqrt{65}$.
8. How many distinct prime divisors does $3^{18} - 2^{18}$ have?
- (a) 3.
 - (b) 4.
 - (c) 5.
 - (d) 6.
 - (e) more than 6.

9. Find the greatest common divisor of $2^{900} + 1$ and $2^{675} + 1$.
- (a) 1.
 - (b) $2^{225} - 1$.
 - (c) 2^{225} .
 - (d) $2^{225} + 1$.
 - (e) $2^{675} + 1$.
10. If $a^3 - b^3 = 24$ and $a - b = 2$, how many possible values are there for $a + b$?
- (a) 0.
 - (b) 1.
 - (c) 2.
 - (d) 3.
 - (e) more than 3.
11. Alice, Bob, Carol, and Dave are rescued from a desert island by a pirate who forces them to play a game. Each of the four, in alphabetical order by first name, is forced to roll two fair six-sided dice. If the total on the two dice is either 8 or 9, the person rolling the dice is forced to walk the plank. The players go in order until one player loses. What is the probability that Dave survives?
- (a) $1/4$.
 - (b) $44/175$.
 - (c) $148/175$.
 - (d) $97/350$.
 - (e) $227/350$.
12. Jase and Willie are playing Risk. Jase rolls one six-sided die while Willie rolls two of them. What is the probability that Jase's roll is greater than or equal to Willie's larger number?
- (a) $1/2$.
 - (b) $11/36$.
 - (c) $5/36$.
 - (d) $91/216$.
 - (e) $31/216$.

13. How many possible quiz-bowl teams with four people (one being a captain) can be chosen from ten people?
- (a) 24.
 - (b) 40.
 - (c) 210.
 - (d) 840.
 - (e) 3,628,800.
14. In an alley between two buildings, there are two ladders. One is 10 feet long, while the other is 11 feet long. They form an x, with one ladder leaning against the base of one building, the other leaning the opposite way, across the alley. The two ladders intersect at a point that is 3 feet above the ground. How far apart are the two buildings?
- (a) 7.246 feet.
 - (b) 4.653 feet.
 - (c) 5.262 feet.
 - (d) 8.503 feet.
 - (e) 6.978 feet.
15. How many distinct sets of four letters can be formed from the word TRIANGLE if we want at least one vowel and do not want to repeat any letters?
- (a) 40.
 - (b) 360.
 - (c) 10.
 - (d) 70.
 - (e) 65.
16. Let a, b and x be positive real numbers. If

$$2 \log_b x = 2 \log_b(1 - a) + 2 \log_b(1 + a) - \log_b \left(\frac{1}{a} - a \right)^2$$

Find x .

- (a) a .
- (b) b .
- (c) a^2 .
- (d) b^2 .
- (e) 1.

17. Let $i = \sqrt{-1}$. Find how many real solutions are there to the following equation.

$$x^5 + (8 - i)x^3 + (18 - 8i)x^2 + (16 - 16i)x + 1312 = 0$$

- (a) 0.
 - (b) 1.
 - (c) 2.
 - (d) 3.
 - (e) 4.
18. Suppose a, b and c are positive real numbers with $c^a = 1.3871$ and $c^b = 1.3882$. If $c > 1$ then to 4 decimal places what is $c^{\frac{a+b}{2}}$?
- (a) 1.3876.
 - (b) 1.3877.
 - (c) 1.3878.
 - (d) 1.3879.
 - (e) 1.3880.
19. There is a list of 71 consecutive positive integers such that the sum of the squares of the first 36 is equal to the sum of the squares of the last 35. Find the last integer in the list.
- (a) 2552.
 - (b) 2553.
 - (c) 2554.
 - (d) 2555.
 - (e) 2556.
20. What is the vertex of the parabola given by $x^2 - 8x + 21$?
- (a) (-4,69).
 - (b) (-5,86).
 - (c) (4,5).
 - (d) (5,6).
 - (e) None of the above.

21. The curves $y = \tan(x)$ (in radians) and $y = x$ intersect an infinite number of times. Let $x_0 = 0$, x_1 be the first positive root of $\tan(x) = x$, x_2 be the second, and so forth. What is $\lim_{n \rightarrow \infty} x_{n+1} - x_n$, rounded to the first decimal place?
- (a) 1.6.
 - (b) 2.9.
 - (c) 3.0.
 - (d) 3.1.
 - (e) Does not exist.
22. A binary operation $*$ is defined on the positive real numbers by $x * y = \sqrt{yx}$. If z is also a real number greater than 1 then $(x * y) * z$ is equal to
- (a) \sqrt{xyz} .
 - (b) $\sqrt{\sqrt{xyz}}$.
 - (c) $\sqrt[4]{xyz^2}$.
 - (d) $\sqrt[4]{x^2yz}$.
 - (e) None of the above.
23. Given the elliptical region by $x^2 + 9y^2 \leq 36$, which of the following points lies outside the given region?
- (a) (1,1).
 - (b) (-3,1).
 - (c) (0,2).
 - (d) (2, -1).
 - (e) (1,2).
24. Consider the following logical premises:
- $q \rightarrow \sim p$
 - $p \vee r$
 - $\sim r$

Which of the following conclusions makes the argument valid?

- (a) $\sim p$.
- (b) q .
- (c) $\sim q$.
- (d) r .
- (e) None of these..

25. Let $x_1 = c$ for some positive number c , and let $x_{n+1} = 2^{x_n - 2}$. What is the set of values of c such that the sequence x_1, x_2, x_3, \dots converges?
- (a) Empty.
 - (b) $(0, 2)$.
 - (c) $(0, 2]$.
 - (d) $(0, 4)$.
 - (e) $(0, 4]$.
26. The ones digit of $(2013)^{2014}$ is
- (a) 0.
 - (b) 1.
 - (c) 3.
 - (d) 7.
 - (e) 9.
27. When $1.\overline{237}$ is written as a rational number in lowest terms, the sum of the numerator and denominator is
- (a) 745.
 - (b) 746.
 - (c) 747.
 - (d) 748.
 - (e) 749.
28. Let $x \neq -1$ be a real number. If $x, 2x + 2, 3x + 3, \dots$ are in a geometric progression then the 4^{th} term is
- (a) 27.
 - (b) -13.5.
 - (c) -12.
 - (d) 12.
 - (e) 13.5.

29. Let r be any real number. Which of the following is always greater than r ?

(a) $2r$.

(b) $3.2(6r^2 - 3) + 9.5$.

(c) $60r^{100}$.

(d) $2.5 \left(\frac{2r^2}{5} + \frac{2}{5} \right)$.

(e) $(r + 1)^3 + (r + 1)^2$.

30. For each real number x define the function $f(x)$ to be the minimum of the numbers in the list $\{3x + 1, x + 1, -2x + 5\}$. What is the absolute maximum value of $f(x)$?

(a) $\frac{4}{3}$.

(b) $\frac{4}{5}$.

(c) $\frac{17}{5}$.

(d) $\frac{7}{3}$.

(e) 1.