

High School Mathematics Contest

Elon University Mathematics Department

Saturday, April 13, 2015

1. If two fair dice are rolled, what is the probability (rounded to the nearest percent) that the sum of the numbers on top is greater than the product?
 - (a) 0%.
 - (b) 6%.
 - (c) 17%.
 - (d) 31%.
 - (e) 50%.
2. A solution to the equation $\log_4(x) + \log_4(5) - \log_4(2) = \frac{3}{2}$ is
 - (a) $\frac{16}{5}$.
 - (b) -1 .
 - (c) $\frac{18}{5}$.
 - (d) $\frac{5}{18}$.
 - (e) $\frac{4}{5}$.
3. Let $f(x) = |x + 2| + 2|x - 2|$. What is the area between the graph of $y = f(x)$ and the x -axis from $x = -3$ to $x = 3$?
 - (a) 33.
 - (b) 35.
 - (c) 37.
 - (d) 39.
 - (e) 41.
4. Each valve A , B , and C , when open, releases water into a tank at its own constant rate. With all three valves open, the tank fills in 1 hour. With only valves A and C open, it takes 1.5 hours. And with only valves B and C open, it takes 2 hours. How long will it take to fill the tank with only valves A and B open?
 - (a) 1.1 hours.
 - (b) 1.2 hours.
 - (c) 1.25 hours.
 - (d) $\frac{4}{3}$ hours.
 - (e) 1.5 hours.

5. Suppose we have three distinct positive numbers x , y , and z such that,

$$\frac{y}{x-z} = \frac{x+y}{z} = \frac{x}{y}.$$

Find the numerical value of x/y .

- (a) 1.
 - (b) $1/2$.
 - (c) $-y/z$.
 - (d) 2.
 - (e) not enough information given.
6. Let $i = \sqrt{-1}$ and x, y be real numbers. Suppose $2x + iy + 2y^2 + 5i = 0$. What is the value of $x^2 + y$?
- (a) 45.
 - (b) 55.
 - (c) 620.
 - (d) 630.
 - (e) Not enough information given to determine $x^2 + y$.
7. How many solutions are there for $\cos(\theta) = 0$, where $0 \leq \theta \leq 2015$?
- (a) 639.
 - (b) 640.
 - (c) 641.
 - (d) 2015.
 - (e) 2016.
8. If the ratio of the surface areas of two cubes is k , then the corresponding ratio of the volumes is
- (a) $\sqrt[3]{k}$.
 - (b) $\sqrt{k^3}$.
 - (c) k^2 .
 - (d) k^3 .
 - (e) None of (a) - (d) are correct.

9. IF the binary operation $*$ is defined on the integers by $a * b = a + b + 1$ then which are the following true.
- I. $*$ is commutative.
 - II. $*$ is associative.
 - III. There exists an integer which is the identity for $*$.
- (a) I only .
 - (b) III only.
 - (c) I and II only.
 - (d) I and III only.
 - (e) I, II and III.
10. When $0.\overline{123}$ is written in rational form in lowest terms the sum of the numerator and denominator is
- (a) 123.
 - (b) 374.
 - (c) 377.
 - (d) 1122.
 - (e) 1131.
11. Segments AD and BE are medians of right triangle ABC , and AB is its hypotenuse. If a right triangle is constructed with legs of the same length as AD and BE , what will be the length of its hypotenuse in terms of AB ?
- (a) $AB\sqrt{2}$.
 - (b) $2AB\sqrt{3}$.
 - (c) $\sqrt{5}AB/2$.
 - (d) $\sqrt{AB^2 + 1}$.
 - (e) $\sqrt{3}AB/4$.
12. Suppose that the graph of $y = f(x)$ has symmetry with respect to both the x -axis and the y -axis, where $f(x)$ is defined for all real numbers x . How many roots does $f(x)/x$ have for $x > 1$?
- (a) 0.
 - (b) 1.
 - (c) 2.
 - (d) 4.
 - (e) An infinite number.

13. Rachel went to the whiteboard and drew a circle of radius r . Inside that circle she drew a circle whose diameter was the radius of the first circle. Inside the second circle she repeats the process. Supposing she can continue this process indefinitely, what is the sum of all the areas of the circles?
- (a) $(2\pi r)^2$.
 - (b) $2\pi r^2$.
 - (c) $4\pi r^2$.
 - (d) $6\pi r^2$.
 - (e) $\frac{4}{3}\pi r^2$.
14. The median of three numbers is 3 while the median of their squares is 16. What is the smallest possible value for the range of the three numbers?
- (a) 7.
 - (b) 8.
 - (c) 9.
 - (d) 16.
 - (e) 25.
15. Let $ABCD$ be a rectangle with coordinates $A(0,0)$, $B(0,3)$, $C(2,3)$ and $D(2,0)$. By rolling the rectangle (no slipping) $ABCD$ 180° clockwise along the x -axis so that vertex B is now on the x -axis, find the area under the curve traced out by the vertex B and the x -axis
- (a) $\frac{17}{8}\pi + 6$.
 - (b) $\frac{17}{4}\pi + 6$.
 - (c) $2\pi + 6$.
 - (d) $\frac{17}{2}\pi + 6$.
 - (e) $13\pi + 6$.
16. $\frac{1}{1 - \sqrt[3]{2} + \sqrt[3]{4}}$ is equivalent to
- (a) $1 + \sqrt[3]{2}$.
 - (b) $\frac{1}{3} - \frac{1}{3}\sqrt[3]{2}$.
 - (c) $-\frac{1}{3} + \frac{1}{3}\sqrt[3]{2}$.
 - (d) $-1 + \sqrt[3]{2}$.
 - (e) $\frac{1}{3} + \frac{1}{3}\sqrt[3]{2}$.
17. An angle equal to $\frac{2}{3}$ of its complement has a measure of
- (a) 27° .
 - (b) 36° .
 - (c) 54° .
 - (d) 72° .
 - (e) 108° .

18. Let $ABCD$ be a parallelogram of area 10 with $AB = 3$ and $BC = 5$. Locate E , F , and G on segments AB , BC , and AD , respectively, with $AE = BF = AG = 2$. Let the line through G parallel to EF intersect CD at H . Find the area of the quadrilateral $EFHG$.
- (a) 5.
 - (b) $\sqrt{3^2 + 5^2}/10$.
 - (c) $10/3$.
 - (d) $3\sqrt{5}/10$.
 - (e) $5/2$.
19. How many polynomials $f(x)$ are there such that $f(f(x)) = 16x + 15$?
- (a) 0.
 - (b) 1.
 - (c) 2.
 - (d) 4.
 - (e) An infinite number.
20. Let P be the product of the first 2015 primes. What is the units digit of P ?
- (a) 0.
 - (b) 2.
 - (c) 4.
 - (d) 5.
 - (e) 8.
21. In base 5, let $x = 0.\overline{31}$. Write x in base 10 as a fraction in lowest terms.
- (a) $2/3$.
 - (b) $3/5$.
 - (c) $2/5$.
 - (d) $1/3$.
 - (e) $1/5$.
22. A 16 inch diameter pizza pie is cut into 8 equally sized sectors. Lauren pulls out one piece. What is the perimeter of the remaining pizza?
- (a) $14\pi - 16$.
 - (b) $16\pi - 16$.
 - (c) $14\pi + 16$.
 - (d) $16\pi + 14$.
 - (e) $16\pi + 16$.

23. Let a and b be distinct positive numbers such that $\log_a(b) = \log_b(a)$. How many possible values can $\log_a(b)$ take?
- (a) 0.
 - (b) 1.
 - (c) 2.
 - (d) 4.
 - (e) An infinite number.
24. Let A, B, C and D be coplanar points, no three of which are collinear. Further suppose that D is in the interior of the triangle ABC . Of the 4 possible triangles, what is the minimum number of obtuse angles?
- (a) 0.
 - (b) 1.
 - (c) 2.
 - (d) 3.
 - (e) 4.
25. Simplify: $\frac{1 + \sec(x)}{\sin(x) + \tan(x)}$
- (a) $\cos(x)$.
 - (b) $\sin(x)$.
 - (c) $\sec(x)$.
 - (d) $\csc(x)$.
 - (e) $\tan(x)$.
26. The average of 8 distinct positive integers is 31. What is the largest value any of these numbers can be?
- (a) 35.
 - (b) 39.
 - (c) 218.
 - (d) 220.
 - (e) 248.

27. Let $\triangle ABC$ be any non-degenerate triangle, with side a opposite A , side b opposite B , and side c opposite C . How many values can the measure of $\angle C$ take such that $\sqrt{a} + \sqrt{b} = \sqrt{c}$?
- (a) 0.
 - (b) 1.
 - (c) 5.
 - (d) 9.
 - (e) An infinite number.
28. Let n be a positive integer. Suppose in base n arithmetic we have $4 \times 5 = 26$. What is the value of $4 + 5$?
- (a) 7.
 - (b) 9.
 - (c) 12.
 - (d) 16.
 - (e) Not enough information to answer the question..
29. Find the sum of all the solutions to the equation $\sin^3(x) + \cos^3(x) = \frac{1}{2}(\sin(x) + \cos(x))$ with $0 \leq x < 2\pi$.
- (a) $\frac{5\pi}{4}$.
 - (b) $\frac{7\pi}{4}$.
 - (c) $\frac{7\pi}{2}$.
 - (d) 4π .
 - (e) $\frac{17\pi}{4}$.
30. How many distinct ways can 5 X's and 4 O's be placed on a 3×3 grid?
- (a) 45.
 - (b) 63.
 - (c) 126.
 - (d) 3024.
 - (e) 15120.